PSE 2025 Grade 8 Problem Set

Instructions: You will have 60 minutes to complete 30 questions. Your answer is the number of problems you get correct. Only answers written on the provided answer sheet will be graded. This is an individual test; anyone caught talking with others will have their score disqualified. You are allowed a pencil/pen/writing utensil and scratch paper, which will be provided. Calculators, compasses, rulers, protractors, formula sheets, and the Internet are not allowed.

Solve as many problems as you can. Good luck, and have fun!

1. What is the largest prime factor of 3570?
2. How many positive integers can fill the blank in the sentence below? "One positive integer is ____ more than twice another, and the sum of the two numbers is 28."
3. A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?
4. A gnome has a collection of 3-legged ants and 5-legged spiders. If he has 20 total bugs and counts 78 legs, how many spiders does he have?
5. 14 ants are trying to get to sugar. If each ant has a 2/7 chance to reach the sugar successfully, what is the expected number of ants that will reach the sugar?
6. At a fairy formal with 30 total fairies, each female fairy dances with 3 male fairies, and each male fairy dances with 2 female fairies. How many female fairies are at the formal?
7. Triangle ABC has $AB = 10$, $AC = 11$, and $BC = 16$. A circle is inscribed in the triangle. What is its radius?
8. Aprameya is biking to his USAMO competition 12 miles away. He starts biking at 6:30 AM at 18 mph. He gets a flat halfway and runs the rest. If the competition starts at 8:02, what is the slowest speed he could run?
9. Tetrahedron $VABC$ has volume 1/10, $VA = 1$, $VB = 2$, $VC = 3$. Points X, Y, Z are on VA, VB, VC such that $VX = 3$, $VY = 2$, $VZ = 1$. Find the volume of tetrahedron $VXYZ$.
10. Bob has a square pyramid hat with edge length 4 placed on top of a spherical head, covering half of his face. What is the radius of Bob's head?
11. Rithik plays peekaboo with 2 other kids. John wins \$9 per win, Bob wins \$13 per win. What is the largest impossible combined sum of John and Bob's winnings?
12. Parallelogram $JKLM$ has 3 vertices at $(0, 0)$, $(5, 0)$, and $(4, 5)$. Find the absolute difference between the areas of the largest and smallest possible parallelograms.
13. Let $\text{LCM}(9, x) = 225$. Find the sum of all possible x.
14. Find the sum of all integers x such that for some integer y, $3xy + 6x + y = 98$.
15. Regular hexagon $ABCDEF$ with side 4 has a square added at AB, with an equilateral triangle attached to the square. Find the distance from the triangle vertex to D.
16. John is tiling a floor that is 8 feet by 10 feet. He has access to 2 in x 2 in, 4 in x 4 in, and 12 x 12 in tiles, which cost \$0.75, \$2.25, and \$24.50, respectively. If John chooses the option that costs him the least money, how much will he pay to tile his floor?

17. After passing USAMO, Aprameya is competing at IMO. There are 6 problems, of which Aprameya solves at least 3 of them. How many different combinations of problems solved does Aprameya have?

18. A splendid number has at least 2 digits and is divisible by the sum of its first and last digit. Find the number of splendid numbers less than 500.

19. Solve for x: $\frac{\log_2 x^2}{\log_4 3} - \frac{3(\log_2 9)(\log_3 x)}{2} = \log_4 81$.

20. A rectangle 5×13 is inscribed in a semicircle with the 13-length side along the diameter. Find the area of the semicircle.

21. Let r, s, t be roots of $P(x) = x^3 - 6x^2 + 11x - 6$. Find $r^2 s + s^2 t + t^2 r$.

22. Find the radius of the largest circle that fits in the gap made by the x and y axes with a circle centered at $(5, 5)$ with a radius 5.

23. How many integers, between 1 and 624 (inclusive), have an odd number of non-zero digits when expressed in base 5?

24. A circle is inscribed in a quadrilateral $ABCD$ such that it is tangent to sides AB, BC, CD, and DA at points P, Q, R, and S, respectively. If $AB = 10$, $BC = 15$, and $CD = 12$, find the length of DA.

25. In triangle $\triangle ABC$, the incircle is tangent to sides AB, BC, and CA at points P, Q, and R, respectively. If $AB = 13$, $BC = 14$, and $CA = 15$, find the radius of the incircle.

26. Nicky is hanging up holiday ornaments on 3 different hooks. She has 10 ornaments, each of a different size. She wants to arrange them such that each hook has at least 1 ornament, and a smaller ornament is never holding up a larger ornament (i.e. after the hook, the ornament sizes are decreasing). How many different ways can she do this?

27. An ant is on a pile of dirt at (0,0) on the coordinate plane. If every second the ant has an equal probability of moving up one unit or right one unit, what's the expected location of the ant after 30 seconds?

28. An ant at (0,0) on the coordinate plane randomly chooses a time in minutes between (0,1) and a distance to travel between (0, 1). (an example would be 0.46 minutes to travel 0.15 units). The ant finishes its last step when the total time has exceeded one minute. What is the probability the ant walks more than one unit?

29. An election occurs where candidate A received 25 votes and candidate B received 20 votes. Consider that the election happened one at a time; each vote was individually tallied. Ignoring the starting tied position where candidates had 0 votes each, what is the probability that candidate A's vote tally was always greater than candidate B's vote tally?

30. A body-centered cubic arrangement is a lattice arrangement of unit cubes, each with some length A. An atom is placed at each corner of the cube, as well as the center of the cube. For an atom P, define the region D to be the set of all points closer to P than any other atom in the lattice. If A = 6, find the value of D.